PHYC 581: High Energy Astrophysics
Fall 2018

Leture 3: 08/27/2018

Energies, Luminosities and Time Scales:

In this lecture, we estimate certain critical numbers that characterize the high-energy emission by compact objects accreting from their environment. These considerations are important for building high-energy telescopes and effectively using them for the signals reaching the Earth from distant sources.

The principal source of power in diverse system, like X-ray binaries and active galactic nuclei (AGN), is the release of gravitational potential energy from mass accreting onto a compact object. Consider a mass m falling onto an object of bigger mass M and radius R. The released gravitional potential energy per unit of falling mass is:

$$\Delta E_{acc} \simeq \frac{GM}{R}$$

To put this in perspective, let us compare ΔE_{acc} with ΔE_{nuc} (i.e., the energy released in the pp-chain of Hydrogen burning) for various objects. We have:

$$\Delta E_{nuc} (4p \rightarrow {}^4He) \simeq 6 \times 10^{18} \text{ erg g}^{-1}$$

For a neutron star, $M \sim 1 M_\odot$ and $R \sim 10$ km, which results in:

$$\Delta E_{acc}(NS) \simeq 10^{20} \text{ erg g}^{-1} \sim 20 \, \Delta E_{nuc}$$

For a white dwarf, $M \sim 1 M_\odot$ and $R \sim 10{,}000$ km, which leads to:

$$\Delta E_{acc}(WD) \simeq 10^{17} \text{ erg g}^{-1} \sim \frac{1}{50} \Delta E_{nuc}$$

If we consider a black hole, we should use the Schwarzshild radius $R_s = \frac{2GM}{c^2}$, whic leads to:

$$\Delta E_{acc}(BH) \simeq 5 \times 10^{20} \text{ erg g}^{-1} \sim 100 \, \Delta E_{nuc}$$

Two additional questions that arise regarding the energy released by gravity are:

(1) What is the total luminosity associated with the process?

(2) How quickly can the emission region vary?

To answer the first question, let us consider a plasma of ionized Hydrogen that is in hydrostatic equilibrium. Ionization assumption is justified in the case of X-ray or γ-ray emission since X-ray and γ-ray photons are energetic enough to ionize Hydrogen. In hydrostatic equilibrium, the inward pull of gravity is balanced by the outward push by radiation.

Photons diffusing outward interact with the electrons and protons. The cross section for scattering goes like $\frac{1}{m^2}$, m being the mass of charged particle, and hence it is much larger for photon scattering off electrons. If we consider Thomson scattering, which is the case if energy of photons is $E \ll m_e c^2$, the total scattering cross section is given by $\sigma_T = 6.65 \times 10^{-24}$ cm^2.

The gravitational force on an electron-proton pair is given by:

$$F_{grav} = \frac{-GM(m_e + m_p)}{r^2} \simeq -\frac{GMm_p}{r^2}$$

The momentum carried by radiation with energy flux F is:

$$\Pi = \frac{F}{c}$$

The force from radiation (assuming 100% absorption of the momentum of scattered photon) is therefore given by:

$$F_{rad} = \frac{F}{c}\sigma_T$$

The two forces cancel each other if:

$$\frac{F}{c}\sigma_T = \frac{GMm_p}{r^2}$$

The flux at a distance r is related to the luminosity

L at that distance according to:

$$F = \frac{L}{4\pi r^2}$$

There fore, in hydrostatic equilibrium we have:

$$\frac{\sigma_T L}{4\pi c} = G M m_p$$

This yields a critical luminosity, called the Eddington limit

L_{edd}, for which the equilibrium holds:

$$L_{edd} = \frac{4\pi c G M m_p}{\sigma_T}$$

Numerically, we have:

$$L_{edd} \simeq 1.3 \times 10^{38} \left(\frac{M}{M_\odot}\right) \text{ erg } s^{-1}$$

For a neutron star with $M = 1.4\, M_\odot$, we have:

$$L_{edd}\,(NS) \sim 10^5\, L_\odot$$

For an AGN black hole with $M = 10^7\, M_\odot$, we have:

$$L_{edd}(AGN) \sim 7 \times 10^{11}\, L_\odot$$

If $L > L_{edd}$, the source will expel nearby matter and quench the process of accretion.

The effective temperature T_{eff} of a blackbody with total power L_{edd} is:

$$T_{eff} = \left(\frac{G M m_p}{R^2 \sigma_T \sigma_B}\right)^{\frac{1}{4}}$$

σ_B: Stefan-Boltzmann constant $= 5.67 \times 10^{-8}\ W m^{-2} K^{-4}$

For a neutron star, we find $T_{eff}(NS) \sim 2 \times 10^7\ K$, and the characteristic photon energy is ~ 1.6 keV (hence X-ray photon).

For a white dwarf, $T_{eff}(WD) \sim 6 \times 10^5\ K$, which results in a characteristic photon energy of ~ 50 eV (hence UV photon).

For a black hole the situation is more complicated because, due to the absence of a hard surface, the gravitational potential energy is released over an extended region.

The photon number flux on the Earth arriving from a source at the center of the galaxy is:

$$F_{ph} = \frac{L_{edd}}{4\pi (8.5\ kpc)^2 \langle \varepsilon \rangle}$$

Here $\langle \varepsilon \rangle$ is the average photon energy, and we have ignored attenuation of the flux by the interstellar medium. For an X-ray source such as a neutron star, $\langle \varepsilon \rangle \sim 1.6\,keV$, which results in:

$F_{ph} \sim 10\ cm^{-2}\ s^{-1}$

The Chandra observatory achieves a sensitivity of about $2\times10^{-6}\ cm^{-2}\ s^{-1}$, which allows it to discover very faint objects.

Now let us turn to the second question in above. For gravity, the variability time scale t_{var} is influenced by the dynamical time scale t_{dyn}. This provides an upper limit on t_{var}. For an element of mass m falling under gravity, t_{dyn} can be estimated as follows. The time that it takes for the mass to fall a distance R (i.e., radius of the object) is:

$$t_{dyn} = \left(\frac{2R}{a}\right)^{1/2} = \left(\frac{2R^3}{GM}\right)^{1/2}$$

Here we use $a = \frac{GM}{R^2}$ for the gravitational acceleration.

For a neutron star, we find:

$t_{dyn}(NS) \sim 10^{-4}$ s

For a white dwarf, we have:

$t_{dyn}(WD) \sim 3$ s

The Doppler shift and light travel time can seriously modify t_{var} inferred by a distant observer.

It is important to note that an instrument should accumulate a reasonable number of photons to measure the incoming flux from typical high energy sources in several seconds, if it is going to detect changes in the emitter's configuration. In the case of a neutron star, integration time is much shorter (a fraction of a millisecond).